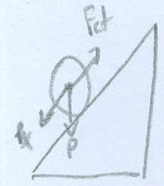
EUF 2011 - 1sem 03/04/2018

Q1.

a)



$x = \phi R$

$\boxed{\dot{x} = \dot{\phi} R}$

force causing rotation, dislocated of the P!

$\tau = I\alpha$

$F_{at} \cdot R = (I + mR^2)\frac{a}{R}$

b) $M\ddot{x} = P_x - F_{at} \Rightarrow M\ddot{x} = Mg\sin\theta - Mg\cos\theta \cdot \mu_e$

$\boxed{\ddot{x} = g\sin\theta - g\cos\theta\,\mu_e}$ $\Rightarrow a = g\sin\theta - (I + mR^2)\frac{a}{R^2}$

$\ddot{x} = \frac{g\sin\theta}{[1 - \frac{1}{R^2}(I + mR^2)]}$

c) $mgh = \frac{mv^2}{2} + \frac{I\omega_f^2}{2}$ (se for conservada) ↳ porque que se não deslizar

Q3.

a) $kr = \frac{mv^2}{r} \Rightarrow mkr^4 = n^2\hbar^2 \Rightarrow \sqrt[4]{\frac{n^2\hbar^2}{mk}} = \boxed{\frac{\sqrt{n\hbar}}{\sqrt[4]{mk}} = r_n}$ $n > 0$

b) $E = \frac{p^2}{2m} + \frac{kr^2}{2} = \frac{mkr^2}{2m} + \frac{kr^2}{2} = kr^2 \Rightarrow \boxed{E_n = k \cdot \frac{n\hbar}{\sqrt{mk}} = \sqrt{\frac{k}{m}} \cdot n\hbar}$

$m^2v^2 = mkr^2$

$\Delta E = E_2 - E_1 = \hbar\sqrt{\frac{k}{m}} \Rightarrow E = h\nu \Rightarrow \boxed{\nu = \sqrt{\frac{k}{m}}}$

c) $p = \frac{h}{\lambda}$ $E_2 = 2\hbar\sqrt{\frac{k}{m}}$;

$E = \frac{p^2}{2m} = \frac{h^2}{\lambda^2 2m} \Rightarrow \lambda = \frac{h}{\sqrt{E2m}} \Rightarrow \boxed{\lambda = \sqrt{\frac{h^2 m}{2\hbar k}} = \sqrt{\frac{m h \pi}{k}}}$ $\hbar = \frac{h}{2\pi}$

d) $n\lambda = 2d\sin\theta$

$n\lambda_1 = 2 \cdot 0{,}8 \cdot 10^{-5}$ ...

Q4.

a) $K_{max} = h\nu_0 - \phi = \underline{0{,}5\,eV}$ $\quad E = \frac{hc}{\lambda} = \frac{4{,}14 \cdot 10^{-15} \cdot 3 \cdot 10^{8}}{4{,}14 \cdot 10^{-7}} = 3\,eV$

b) nada, a energia cinética só depende da freq.

c) $\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$ $\qquad \underline{\lambda, E, p} \begin{cases} \lambda', E', p_1 \\ K, p \end{cases}$

$E' = \frac{E}{2}$

$\lambda' = \lambda_c(1 - \cos\theta) + 2\lambda_c = \lambda_c(3 - \cos\theta)$ $\qquad E = E' + K$

$\frac{E}{2} = K \Rightarrow E = 2K$

$\lambda' = \frac{hc}{E'} = \frac{2hc}{E} = \frac{2hc}{2\lambda_c} = \frac{hc}{\lambda_c}$

$\frac{hc}{\lambda_c^2} = 3 - \cos\theta \Rightarrow \theta = \cos^{-1}\left(3 - \frac{hc}{\lambda_c^2}\right)$

d) $K = \frac{E}{2} = \boxed{\frac{m_0 c^2}{2}}$ $\qquad p = \frac{m_0 c}{2}$ ...

$p^2 = \frac{K^2}{c^2} + 2Km_0 \Rightarrow \frac{m_0^2 c^2}{2} + m_0^2 c^2 \Rightarrow p = \sqrt{\frac{3m_0^2}{2}c^2} = \boxed{m_0 c\sqrt{\frac{3}{2}}}$

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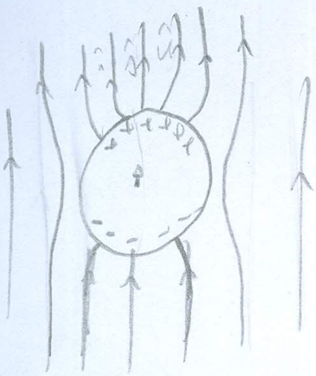
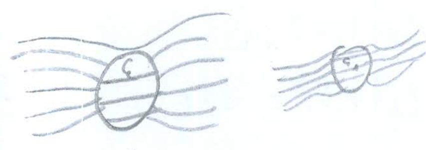
Q6. a)



eletrostática equilíbrio. As cargas vão se mover até que o campo elétrico fique perpendicular. O campo induz cargas na superfície do material e ele terá as cargas negativas e positivas nos pontos. Dentro do condutor o campo é nulo. A carga induzida vai distorcer o campo

d)

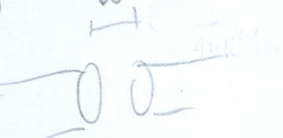


$\varepsilon > \varepsilon_1$

b) $E = E_0 \hat{z}$

$V = -E_0 z + C$

Ver Q7



Q8. a) $\int_{-\infty}^{\infty} A^2 e^{-\frac{2m\omega}{2\hbar}x^2}\, dx = 1 \ldots$

$A = \sqrt[4]{\frac{m\omega}{\pi\hbar}}$; $B = \sqrt[4]{\frac{4m^3\omega^3}{\pi\hbar^3}}$

b) joga ψ_0 na eq. Sch. $\Rightarrow E_0 = \frac{\hbar\omega}{2}$

c) $\langle x \rangle = \int_{-\infty}^{\infty} \psi^*(x)\, x\, \psi\, dx$ temos nos dois só funções ímpares $\therefore \int_{-\infty}^{\infty} = 0$

$\frac{d\langle x \rangle}{d\beta} = 0$; $\frac{d^2\langle x \rangle}{d\beta^2}$ (parabola) < 0 $\beta +$

d) $\psi(x,t) = e^{-\frac{i E_0 t}{\hbar}} \psi_0 + e^{-\frac{i E_1 t}{\hbar}} \psi_1 \Rightarrow \langle x \rangle = \int \psi^*(x,t)\, x\, \psi(x,t)\, dx$

Q9. a)

$$L_+ = L_x \pm iL_y \Rightarrow \boxed{L_x = \frac{L_+ + L_-}{2}}$$
$$L_- = L_x - iL_y$$

$$L_\pm Y_{lm} = \hbar\sqrt{l(l+1) - m(m\pm 1)}\; Y_{l(m\pm 1)}$$

$l=1 \Rightarrow m = -l, 0, l$

$$L_+|1,-1\rangle = \sqrt{2}\hbar\,|1,0\rangle$$
$$L_+|1,0\rangle = \sqrt{2}\hbar\,|1,1\rangle \Rightarrow L_+ \begin{bmatrix} \langle 1,-1|L_+|1,-1\rangle & \langle 1,0|L_+|1,-1\rangle & \langle 1,1|L_+|1,-1\rangle \\ & & \\ & & \end{bmatrix}$$
$$L_+|1,1\rangle = 0\,|1,2\rangle$$

$$L_+ = \begin{bmatrix} 0 & \sqrt{2}\hbar & 0 \\ 0 & 0 & \sqrt{2}\hbar \\ 0 & 0 & 0 \end{bmatrix} \qquad L_- = \begin{bmatrix} 0 & 0 & 0 \\ \sqrt{2}\hbar & 0 & 0 \\ 0 & \sqrt{2}\hbar & 0 \end{bmatrix}$$

$$L_-|1,-1\rangle = 0\,|1,-2\rangle$$
$$L_-|1,0\rangle = \sqrt{2}\hbar\,|1,-1\rangle$$
$$L_-|1,1\rangle = \sqrt{2}\hbar\,|1,0\rangle$$

$$\boxed{L_x = \frac{L_+ + L_-}{2} = \frac{\sqrt{2}\hbar}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$

b) $$\begin{vmatrix} -\lambda & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & -\lambda & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\lambda \end{vmatrix} = 0 \Rightarrow -\lambda^3 + \hbar^2\lambda \Rightarrow \lambda(-\lambda^2 + \hbar^2) = 0$$

$$\boxed{\lambda = 0,\ \lambda = \pm\hbar}$$

c) $\lambda = \hbar$

$$\frac{\hbar}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \cancel{\hbar} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \Rightarrow \frac{b}{\sqrt{2}} = a$$

$$\frac{a+c}{\sqrt{2}} = b \Rightarrow c = a$$

$$\begin{bmatrix} \frac{b}{\sqrt{2}} \\ b \\ \frac{b}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{b}{\sqrt{2}} & b & \frac{b}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{b}{\sqrt{2}} \\ b \\ \frac{b}{\sqrt{2}} \end{bmatrix} = 1$$

$$= 2|b|^2 = 1 \Rightarrow b = \frac{1}{\sqrt{2}} \Rightarrow |L_{x+}\rangle \begin{bmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix}$$

d) $L_z = \begin{bmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{bmatrix} \Rightarrow |L_{z+}\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \langle L_{z+}|L_{x+}\rangle = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix} = \frac{1}{2}$

$a\hbar = a\hbar$

$-\hbar c = \hbar c$

$$\boxed{P(L_{z+}) = \frac{1}{4}}$$

$\lambda = 0$

$$L_z \begin{bmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0 \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$a\hbar = 0$

$-\hbar c = 0$

$0 = 0$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix} \Rightarrow P(L_{z0}) = \frac{1}{2}$$

$\lambda = -\hbar$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow P = \frac{1}{4}$$

Q6.

a) $W = \int_A^B p\,dV = \int_V^{2V} \frac{nRT}{V}\,dV = \underline{nRT\ln 2}$

b) $dU = dQ - dW$ $\quad dT = 0$ porque está constantemente em contato com o reservatório

$dU = 0 \quad dU = C_V dT$

$dQ = dW \Rightarrow \boxed{Q = nRT\ln(2)}$

c) $Q = T\Delta S$

$\boxed{\Delta S_g = nR\ln 2}$ p/ o gás

$Q_g + Q_r = 0$ reservatório

$Q_g = -Q_r \Rightarrow \boxed{\Delta S_R = -nR\ln 2}$

$\Delta S_T = 0$ (processo reversível)

d) $\Delta S > 0$

$\Delta W = 0$ (exp livre)

$\Delta Q = 0$

$dU = 0$

$\Delta S_g = nR\ln 2$

$S_V = 0$

$\boxed{\Delta S_T = nR\ln 2}$